Angular momentum transport in accretion

Prateek Sharma, IISc
Accretion

Two major classes of objects in astrophysics:

pressure supported, e.g. stars
rotation supported, e.g. disks

formed because collapsing matter lost energy but not angular momentum

\[ \frac{dp}{dr} \approx -\rho g \]

\[ \Omega^2 R \approx g_R \]
ALMA disks

TW Hydra  V883 Ori  HD 163296

HL Tau  Elias 2-27  HD 142527

sub-mm emission from dust in protoplanetary disks ~ 100 AU
Scenario for star- and planet formation

Cloud collapse

Protostar with disk

Formation planets

Planetary system

Slide courtesy: Jack Lissauer
Protoplanetary disk
Equilibrium structure

gravity dominated by central object

\[ \frac{GM}{R^2} \approx \Omega^2 R \]  
radial force balance \implies Keplerian flow

\[ \Omega_K = \sqrt{\frac{GM}{R^3}} \propto R^{-3/2} \]

\[ \frac{1}{\rho} \frac{dp}{dz} \approx -\frac{GMz}{R^3} = -\Omega_K^2 z \]
vertical HSE \implies \[ H \approx c_s / \Omega_K \]

\[ H/R \approx c_s / v_K \ll 1 \]

dynamical equilibrium & slow radial accretion of matter
radial & vertical dynamics decoupled \implies 1-D solutions vs.
R of z-integrated quantities
Energy considerations

total energy per mass for an orbiting particle = \(-\frac{GM}{2R}\)

KE in rotation = \(\frac{GM}{2R}\)

gravitational PE = \(-\frac{GM}{R}\)

energy per accreted mass available as heat & light

accretion power generated from R to R + ΔR:

\[ \frac{GM}{2R} \Delta \left( \frac{-1}{2R} \right) = \frac{GM \dot{M}}{2R^2} \Delta R \]

total power released:

\[ \frac{GM \dot{M}}{2R_{in}} \]
Accretion power

\[ \frac{GM \dot{M}}{2R_{\text{in}}} = \eta \dot{M} c^2 \]

higher efficiency for compact accretor

\[ \eta = \frac{GM}{2R_{\text{in}} c^2} = \frac{v_{\text{esc}}^2}{4c^2} = \frac{R_{\text{Sch}}}{4R_{\text{in}}} \]

\[ 4p \rightarrow ^4\text{He} + 2e^+ + 2\nu_e + 27 \text{ MeV} \]

\[ \eta_{\text{nuc}} \approx 27 \text{ MeV}/4 \text{ GeV} = 0.007 \]

no wonder most powerful sources due to accretion!

<table>
<thead>
<tr>
<th></th>
<th>( R_{\text{in}} )(km)</th>
<th>( M(M_\odot) )</th>
<th>( R_{\text{in}}/R_{\text{Sch}} )</th>
<th>( \eta )</th>
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</thead>
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<tr>
<td>protostar</td>
<td>10^6</td>
<td>1</td>
<td>3x10^5</td>
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<tr>
<td>WD</td>
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<td>1</td>
<td>3</td>
<td>0.1</td>
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<tr>
<td>BH</td>
<td>( \frac{9M}{M_\odot} )</td>
<td>10, 10^{-6-9}</td>
<td>3</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Manifestation

~1 Mpc

quasar 3C-273 in a galaxy radiative power

radio galaxy, Cyg A
mechanical power
synchrotron radiation
from relativistic e-\textit{s}

~10 kpc
Flow Temperature

Two estimates:

- virial applicable for non-radiative disks
  \[ k_B T_{\text{virial}} \equiv \frac{GMm_p}{3R} \sim 0.1m_p c^2 \sim 10^{12}\text{K} \]

- geometrically thin, optically thick disk, a local BB disk

T \sim 1\text{ keV} \text{ for maximally accreting 10 solar mass BH, emits X-rays}
Angular momentum transport

**Key Q:** how does matter lose angular momentum and fall in? “molecular viscosity” negligible, need turbulent transport, **but how?**

Linear response: $$w^2 = \frac{k^2}{k^2} \kappa^2, \quad \kappa^2 = \frac{2\Omega}{R} \frac{dl}{dR}$$

$$l = \sqrt{GMR}$$ stable inertial waves
or epicycles oscillations

a perturbed fluid element preserves angular momentum
& **hydro Keplerian disks are Rayleigh stable**
(to local axisymmetric modes)

Hydro Keplerian flow does not naturally break into turbulence
Linear MHD instability

local axisymmetric MHD $\Rightarrow$ MRI
magnetorotational instability
works for ionized flows
results in MHD turbulence & transport
verified by local & global MHD sims [DNS]

magnetic fields behave like stretched strings

originally worked out by Velikhov & Chandrasekhar
Linear MHD instability

\[ \ddot{\xi}_R - 2\Omega \dot{\xi}_\phi = -\left( \frac{d\Omega^2}{d \ln R} + (k \cdot u_A)^2 \right) \dot{\xi}_R , \]

\[ \ddot{\xi}_\phi + 2\Omega \dot{\xi}_R = -(k \cdot u_A)^2 \dot{\xi}_\phi . \]

[Balbus & Hawley 1991]
MHD sims
local shearing boxes
Global simulations

[movies by Prasun Dhang]
Turbulent transport

Pipe flow: stream-wise turbulent momentum transport => flattening of velocity profile

Turbulent Reynolds stress modifies mean flow
Angular momentum eq.

inviscid, $\phi$-averaged angular momentum equation

$$\frac{\partial}{\partial t} (\rho \langle u_\phi \rangle R) + \frac{1}{R} \frac{\partial}{\partial R} \left[ R^2 \left( \langle \rho u_\phi u_R - \frac{B_\phi B_R}{4\pi} \rangle \right) \right] + \frac{\partial}{\partial z} \left( R \langle \rho u_\phi u_z - \frac{B_\phi B_z}{4\pi} \rangle \right) = 0$$

in SS, integrating over $z$, angular momentum flow rate is const.

$$\Sigma \equiv \int_{-H/2}^{H/2} \rho dz$$

$$\dot{M} \equiv 2\pi R \Sigma \langle u_R \rangle$$

$$R^2 \left[ \Sigma \langle u_R \rangle \langle u_\phi \rangle + \langle \Sigma u_{R1} u_{\phi1} - H \frac{B_R B_\phi}{4\pi} \rangle \right]$$

Reynolds & Maxwell stress

$$W_{R\phi} = \frac{\dot{M} l \left( 1 - \frac{l_*}{l} \right)}{2\pi R^2 H}$$

non-zero turbulent stress needed for mass accretion!
Angular momentum

specific angular momentum \( l = \sqrt{GMR} \propto R^{1/2} \)

increases with \( R \); since friction is an internal force only small mass can carry away most angular momentum

\[ \dot{M}_{\text{out}} \sim \dot{M}_{\text{in}} \left( \frac{R_{\text{in}}}{R_{\text{out}}} \right)^{1/2} \]

viscous heating \( \rho \nu_{\text{turb}} \left( \frac{d\Omega}{d \ln R} \right)^2 \) raises \( T \)

disk turbulence not necessary magnetized wind can carry away \( L \)

MHD looks critical for accretion. Is it?
Mean field approach \textit{à la} Reynolds

average density w. poloidal velocity streamlines

mean flow caused by turbulent transport of ang. mom.

[Image courtesy: Prasun Dhang]
disk temperature

\[ T \sim 1 \text{ keV} \left( \frac{M}{10 \ M_\odot} \right)^{1/4} \left( \frac{\dot{M}}{10^{-8} \ M_\odot \text{yr}^{-1}} \right)^{1/4} \left( \frac{r}{100 \ \text{km}} \right)^{-3/4} \]

gives \sim 100 \text{ K} \text{ for } 1M_\odot \text{ & } r=1 \text{ AU}

additional heating due to heating by stellar light

\[ \frac{n_i n_e}{n_H} = \frac{(2\pi m_e k_B T)^{3/2}}{h^3} e^{-\chi/k_B T} \]

thermal ionization essentially gives \( x_e \sim 0 \)

porto-planetary disks (PPDs) are completely neutral

non-thermal ionization due to X-ray, CRs, radioactivity gives \( x_e \sim 10^{-13} \)
Is hydro turbulence possible?

- substantial mass in MRI-inactive dead zones

Can hydro mechanisms operate?

- various non-ideal MHD effects: resistivity, ambipolar diffusion, Hall term
What abt hydro transport?

so common for laminar flow to break into turbulence at large Re

pipe flow linearly stable for all Re; transition observed at Re~2000
sensitive to roughness, vibrations, etc.

why shouldn’t this happen in Keplerian flows where Re>10^{10}?

stabilizing role of (fast) epicyclic oscillations!
Various hydro ideas

transient growth due to non-normality of eigenvectors
transient growth by orders of magnitude
before eventual decay, nonlinearity may take over before this!

subcritical transition to turbulence

Hydrodynamic Stability Without Eigenvalues

Lloyd N. Trefethen, Anne E. Trefethen, Satish C. Reddy, Tobin A. Driscoll
Other hydro instabilities

- driven by vertical rotation gradient $\frac{\partial \Omega}{\partial z}$
- angular momentum transport due to vertical convection
- baroclinic instability driven by $\nabla p \times \nabla \rho$
- Rossby-wave instability, ...

none of hydro mechanisms unanimously accepted by community

BUT PPDs must accrete: layered accretion, MHD winds?

hard problem with lot of observations!
Taylor-Couette Experiments

Rayleigh stability line
\( r_1^2 \Omega_1 = r_2^2 \Omega_2 \)

Linearly unstable
well-studied
Taylor vortices

\( \Omega_1 \)

quasi-
Keplerian
flows

\( \frac{\partial \Omega}{\partial r} < 0 \)

\( \frac{\partial (r^2 \Omega)}{\partial r} < 0 \)

Solid body line
\( (\Omega_1 = \Omega_2) \)

\( \Omega_2 \)

[slide courtesy H. Ji]
Experiments? [Ji et al. 2006]

- Independently rotating endcap rings to prevent Ekman flows
- Quasi-Keplerian rotation
- Small fluctuations up to $Re \sim 10^6$
Are hydro disks stable?

DNS confirm the role of Ekman boundary layers; [Avila 2012]

what happens for Re~10^{15}?

need well-controlled experiments

Keplerian: 1.33
Summary & Future

• angular momentum transport problem

• turbulent transport: MRI mechanism for ionized flow

• what about neutral disks? linear instabilities, nonlinear mechanisms, epicyclic stabilization

• Taylor-Couette flows at Re~10^{15}

• need experiments with controlled axial boundaries

• higher resolution local & global MHD sims with realistic microphysics (chemistry, thermodynamics, radiation, …)

• dynamo & self sustained B-fields; role of large scale fields & outflows

Thank you