Phase Diagram of Under-doped Cuprate Superconductors

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Classical/Conventional Superconductors

1. With the exception of a few systems discovered since 1975 which have $T_c$'s up to $\sim 40 K$, the transition temperature $T_c$ is always below 25 K.

2. The normal state of the metal or alloy in question appears to be well described by Landau Fermi-liquid theory.

3. The superconducting phase does not occur in proximity to other kinds of phase transition.

4. The order parameter is (universally believed to be) of the simple s-wave type.

5. The behavior seems consistent with the hypothesis that the dominant role in the formation of Cooper pairs is, as postulated in the BCS theory, an attractive interaction resulting from the exchange of virtual phonons.

6. $Sn$ (3.72), $Hg$ (4.15), $Pb$ (7.19), $Nb_3Ge$ (23.2) and $MgB_2$ (39) ...
And in addition classical superconductors have two lesser described characteristics are that

1. The crystal structure is relatively simple, and in particular not strongly anisotropic.

2. In the case of alloys, superconductivity is not particularly sensitive to the chemical stoichiometry.
Exotic/Unconventional Superconductors

- Not in the class of classical superconductors and it fails to satisfy at least one of the above-mentioned conditions. In addition, there is reason to believe that it may not be well described by the BCS theory.
- Contains a number of different groups, all discovered since 1975 and examples are the heavy-fermion superconductors, the organics, the ruthenates, the alkali-doped fullerenes and “the cuprate” (high-temperature) superconductors.
Cuprates are the only class of materials to date in which superconductivity has been (reproducibly) observed to occur above 50$K$.

They do not occur in nature, and have been synthesized in the laboratory only since the early 1980’s. There are several hundred different cuprate materials, of which a substantial fraction show superconductivity at temperatures of the order of 100$K$.

However, the value of $T_c$ is usually critically dependent on the chemical stoichiometry, and there exists a non-negligible subclass of cuprates which apparently cannot be made superconducting under any conditions—a feature which is not often commented on, but which may hold an important clue to the origin of superconductivity in these materials.
The Cuprates-Composition

In Conventional form the composition of the cuprates are directly specify in terms of the chemical composition, e.g. \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \) and \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta} \) (where \( x \) and \( \delta \) are positive fractional quantities).

The general formula of cuprates can be written in the following form:

\[
(C\text{CuO}_2)_nA_{n-1}X
\]

where \( n \) is a positive integer, \( A \) is an alkaline earth or rare earth element (or \( Y \)) and \( X \) is an arbitrary collection of elements, possibly including \( Cu \) and/or \( O \).
<table>
<thead>
<tr>
<th>Common name</th>
<th>Formula</th>
<th>$n$, $A$, $X$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSCO</td>
<td>$La_{2-x}Sr_xCuO_4$</td>
<td>1, $-$, $La_{2-x}Sr_xO_2$</td>
<td>Original “high-$T_c$” material</td>
</tr>
<tr>
<td>YBCO</td>
<td>$YBa_2Cu_3O_{6+\delta}$</td>
<td>2, $Y$, $Ba_2CuO_{2+\delta}$</td>
<td>Probably most studied</td>
</tr>
<tr>
<td>BSSCO</td>
<td>$Bi_2Sr_2CaCu_2O_8$</td>
<td>2, $Ca$, $Bi_2Sr_2O_4$</td>
<td>Best for ARPES, EELS</td>
</tr>
<tr>
<td>HgBCO</td>
<td>$HgBa_2Ca_2Cu_3O_8$</td>
<td>3, $Ca$, $HgBa_2O_2$</td>
<td>Highest $T_c$ to date</td>
</tr>
<tr>
<td>NCCO</td>
<td>$Nd_{2-x}Ce_xCuO_4$</td>
<td>1, $-$, $Nd_{2-x}Ce_xO_2$</td>
<td>Electron-doped</td>
</tr>
<tr>
<td>$\infty$-layer</td>
<td>$Sr_xCa_{1-x}CuO_2$</td>
<td>$\infty$, $Ca(Sr)$, $-$</td>
<td>No Charge-reservoir group</td>
</tr>
</tbody>
</table>

**Table:** Some important superconducting cuprates
The general formula of cuprates reflects the characteristic crystal structure of the cuprates. The “naive” unit cell (i.e. the cell corresponding to a formula unit) consists of a group (“multilayer”) of \( n \) \( CuO_2 \) planes, spaced by \( (n-1) \) layers of the “spacer” or “intercalant” element \( A \) and separated from the next plane or set of planes by the “charge reservoir” unit \( X \).
Figure: Crystal Structure of Cuprate $Bi_2Sr_2CaCu_2O_{8+\delta}$. 
The Cuprates-Phase Diagram

Figure: Phase diagram of Cuprates.
The Cuprates—What do we know for sure about cuprates superconductivity?

- Superconductivity in the cuprates is due to formation of Cooper pairs.
- The principal locus of superconductivity in the cuprates is the $CuO_2$ planes.
- The order parameter is a spin singlet.
- The orbital symmetry of the order parameter is $d_{x^2−y^2}$.
- The formation of Cooper pairs takes place independently within different multi-layers.
- The electron-phonon interaction is not the principal mechanism of the formation of Cooper pairs.
- The size of the Cooper pairs in the $ab$-plane lies in the range 10-30Å.
- The Cooper pairs are formed from time-reversed states.
Is the normal state of the cuprates well described by (some variant of) the traditional Landau Fermi-liquid theory?

Should the anomalous properties of the $S$ state be regarded as reflecting those of the $N$ state, or rather vice versa, i.e. are the $N$-state properties anomalous because of the occurrence of high-temperature superconductivity? In particular, should the “pseudogap” regime be regarded as an extension of the $S$ regime?
Related to the above, is there any “hidden” phase transition somewhere in the neighborhood of the “crossover” line $T^*(x)$? If so, what is the nature of the phase which occurs below this line? Does it correspond to the spontaneous breaking of some symmetry (other than the $U(1)$ symmetry which is often said to be “broken” by the onset of Cooper pairing), and if so, which one?

Is the two-dimensional layered structure of the cuprates an essential ingredient in their high-temperature superconductivity?

Related to the last question, why does $T_c$ increases with $n$ in homologous series up to $n = 3$?
The Cuprates—Some key questions contd..

- Does the long-range part of the Coulomb interaction play a significant role in cuprate superconductivity?

- Why do cuprates having very similar CuO$_2$-plane characteristics (e.g. the Hg and Tl series) nevertheless have substantially different (maximum) values of $T_c$?

- Why, despite the fact that the $ab$-plane properties of the cuprates appear to be at least qualitatively “universal”, does the $c$-axis $N$-state resistivity behave qualitatively differently for different systems?
Most current theoretical approaches to the problem of cuprate SC start from a “model”, that is, postulation of some form of effective “H” which describes the “low-energy” states of the many-body system in at least a qualitatively correct way. Is it obvious that any approach of this type will work for the cuprates?

Is the current maximum value of $T_c$ (150-160K, depending on the exact definition) the theoretical upper limit for any material of the cuprate class? Irrespective of this, are there other classes of materials which would show even higher $T'_c$s, perhaps up to or above room temperature?
Phase coherence in underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$

- In layered high $T_c$ superconductors (HTSC), the density of paired electrons per layer, $N_s$, determine the phase stiffness (energy cost to produce spatial variations in phase within a layer).
- Superconductivity persists upto:
  \[ k_B T \approx k_B T_\theta = N_s \frac{\hbar^2}{m^*} \]
- In conventional SC at low temperature, Pair binding energy $\ll k_B T_\theta$. Transition from SC state to NS occurs because of unbinding of cooper pairs.
- In HTSC, Pair binding energy $\gg k_B T_\theta$. In underdoped (UD) regime this opposite situation is more profound. In UD regime a state with paired electrons and short range phase correlations between $T_c$ and $T^*$ - Pseudo Gap state.
Phase coherence in underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ contd..

- A high-frequency technique (Time-domain transmission spectroscopy) is used to capture the short-time scale dynamics.
- For $T < T_c$, 
  \[ \sigma(\omega) = i \sigma_Q(k_B T_\theta / \hbar \omega) \]
  where $\sigma_Q = e^2 / \hbar d$, is the quantum conductivity of a stack of planar conductor with inter-layer spacing $d$.
- For $T > T_c$, with short range phase correlations, $\sigma(\omega) =$?
Figure 1 The complex conductivity $\sigma$ measured at 100 GHz, as a function of the temperature $T$. The real part, $\sigma_r$, is multiplied by five for ease of comparison with the imaginary part $\sigma_2$. The real part is comprised of a peak near $T_c$ superposed on a smooth background. We interpret the peak as the contribution from a partially coherent superfluid and the background as the contribution from quasiparticles.
Figure 2 The dynamic (frequency dependent) phase-stiffness temperature, $T_\theta(\omega)$ as a function of temperature $T$. Data are shown for two samples, one with $T_c = 33$ K (left side) and the other with $T_c = 74$ K (right side). The three curves for each sample correspond to measurement frequencies of 100, 200 and 600 GHz. The family of curves identify a crossover from frequency-independent to frequency-dependent phase stiffness. The dashed line shows that the crossover corresponds to the KTB condition for two-dimensional melting, that is, when the phase stiffness and the temperature are related by $T_\theta = \frac{(8/\pi)T}{\omega}$. 

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Phase Diagram of Under-doped Cuprate Superconductors
Figure 4 The phase correlation time $\tau = \Omega^{-1}$ and bare phase stiffness $T_\theta^0$ found from the scaling analysis of conductivity data are plotted as a function of temperature. The main panel compares the bare stiffness, shown as filled circles, with the dynamic phase stiffness at 150 GHz (open squares) and at 400 GHz (open diamonds). The inset shows $\tau$ on a semi-log plot. The hatching defines a region where $\tau$ is less than the lifetime of carriers in the normal state, $\hbar/k_B T$. When the data points reach this region, phase fluctuations of the superconducting condensate become indistinguishable from the ballistic dynamics of normal electrons.
Figure: Geometry of the Nernst experiment in the vortex liquid state. Vortices (disks with vectors) flow with velocity $\vec{v}$ down the gradient $- \nabla T \parallel \hat{x}$. Phase slippage induces a dc signal $E_y$ that is antisymmetric in $H$. 

Vortices/Vortex-like excitations in underdoped $La_{2-x}Sr_xCuO_4$

- A thermal gradient is applied to the sample in a magnetic field. Vortices are detected by the large transverse electric field produced as vortices diffuse down the thermal gradient—the Nernst effect.
- The Nernst coefficient is given as:

$$\nu = E_y / (B \nabla T)$$
Figure 4 Contour plot of $(\nu - \nu_n)$ versus $x$ in the phase diagram of LSCO. The contour plot displays how high in $T$ the vortex-like excitations extend for each value of $x$. The upper solid line $T_{\text{onset}}$ is the contour set by our resolution. The pseudogap $T^*$ estimated from heat capacity is about a factor of two larger than $T_{\text{onset}}$. Values of $T_c$ in our samples (circles) match the $T_c$ line (lower solid line) from Takagi et al. We note that the $T_c$ line is roughly similar to the contour line $\nu = 1 \mu V/KT$. 

Figure: The $T$ dependence of $\nu$ in very $UD\ LSCO$ with $x = 0.03$ (open squares), $x = 0.05$ (triangles) and $x = 0.07$ (open circles).
Scheme for theoretical calculations:

- Employ Fluctuation exchange approximation (FLEX) to calculate dynamical phase stiffness $n_s(\omega)/m^*$, where $n_s(\omega)$ is generalization of SFD for finite frequencies.

- For “$\omega=0$ (static case)”, $n_s$ starts to deviate from 0, where “cooper pairs” starts to form and this temperature is identified as $T_c^*$. 

- Phase stiffness obtained from FLEX is used as input for BKT theory with transverse phase fluctuations to get renormalized $n_s^R < n_s$. 

- For “$\omega \neq 0$ (dynamic case)”, dynamical BKT theory is used with phase fluctuations to find the renormalized phase stiffness $n_s^R(\omega)/m^*$. 
Figure 1 Schematic structure of the infinite-layer compound CaCuO$_2$. This is the ‘parent structure’ of the layered high-temperature superconductors. The structure is tetragonal, and consists of CuO$_2$ planes and cationic Ca planes, alternately stacked along the c-axis.
Figure 5 The superconducting phase diagram of CaCuO$_2$. A maximum $T_c$ of 89 and 34 K is obtained at optimal hole and electron doping, respectively. Doping level $x$ is defined as number of charge carriers per CuO$_2$ unit.
The Hamiltonian and Formalism

- One band Hubbard Hamiltonian,
  \[ \mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{corr} = -\sum_{\langle ij \rangle \sigma} t_{ij} \left( c_{i\sigma}^\dagger c_{j\sigma} + h.c. \right) + U \sum_i n_{i\uparrow} n_{i\downarrow} \]

- Now in conserving FLEX one electron self-energy \( \Sigma \) is given by the functional derivative of a generating function \( \Phi \), which is related to free energy \( wrt \) dressed Greens function,
  \[ \Sigma = \frac{\delta \Phi[\mathcal{H}]}{\delta G} \]

- Dyson equation gives:
  \[ G^{-1} = G_0^{-1} - \Sigma \]

- The dressed Greens functions are used to calculate the charge and spin susceptibilities.
The Hamiltonian and Formalism contd..

- The quasiparticle self-energy components $\mathcal{X}_\nu (\nu = 0, 3, 1)$ \textit{wrt} Pauli matrices $\tau_\nu$ in the “Nambu representation”, i.e.
  
  $\mathcal{X}_0 = \omega (1 - Z)$ (renormalization)
  
  $\mathcal{X}_3 = \xi$ (energy-shift)
  
  $\mathcal{X}_1 = \phi$ (gap parameter), are given by:

$$
\mathcal{X}_\nu (\vec{k}, \omega) = \frac{1}{N} \sum_{\vec{k}'} \int_0^\infty d\Omega \left[ P_s (\vec{k} - \vec{k}', \Omega) \pm P_c (\vec{k} - \vec{k}', \Omega) \right] 
\times \int_{-\infty}^\infty d\omega' I (\omega, \Omega, \omega') A_\nu (\vec{k}', \omega')
$$

where $+ \rightarrow \mathcal{X}_0, \mathcal{X}_3$ and $- \rightarrow \mathcal{X}_1$
The Hamiltonian and Formalism contd..

1. \( I(\omega, \Omega, \omega') = \frac{f(-\omega') + b(\Omega)}{\omega + i\delta - \Omega - \omega'} + \frac{f(\omega') + b(\Omega)}{\omega + i\delta + \Omega - \omega'} \)

and

\[ A_\nu(\vec{k}, \omega) = -\frac{1}{\pi} \text{Im} \frac{a_\nu(\vec{k}, \omega)}{D(\vec{k}, \omega)} \]

2. With \( a_0 = \omega Z \), \( a_3 = \epsilon \vec{k} + \xi \), \( a_1 = \phi \), \( D = (\omega Z)^2 - [\epsilon \vec{k} + \xi]^2 - \phi^2 \)

and \( \epsilon \vec{k} = 2t(2 - \cos(k_x) + \cos(k_y) - \mu) \)

3. The \( f \) and \( b \) are Fermi and Bose distribution functions respectively.

4. Furthermore, band filling is given by,

\[ n = \frac{1}{N} \sum_{\vec{k}} n_{\vec{k}} \]

with \( n_{\vec{k}} = \int_{-\infty}^{\infty} d\omega f(\omega) N(\vec{k}, \omega) \)
The Hamiltonian and Formalism contd..

- The interactions due to charge and spin fluctuations are given as,
  \[ P_s = \left( \frac{1}{2\pi} \right) U^2 I_m (3\chi_s - \chi_{s0}) \] with \( \chi_s = \frac{\chi_{s0}}{1-U\chi_{s0}} \),
  and,
  \[ P_c = \left( \frac{1}{2\pi} \right) U^2 Im (3\chi_c - \chi_{c0}) \] with \( \chi_c = \frac{\chi_{c0}}{1-U\chi_{c0}} \).

where, \( I_m\chi_{s0,c0}(\vec{q}, \omega) = \left( \frac{\pi}{N} \right) \int_{-\infty}^{\infty} d\omega' \left[ f(\omega') - f(\omega' + \omega) \right] \) 

\[ \times \sum_{\vec{k}} \left[ N(\vec{k} + \vec{q}, \omega' + \omega) N(\vec{k}, \omega') \pm A_1(\vec{k} + \vec{q}, \omega' + \omega) A_1(\vec{k}, \omega') \right] \]

with, \( N(\vec{k}, \omega) = A_0(\vec{k}, \omega) + A_3(\vec{k}, \omega) \).
For $\omega=0$ (static case) and without phase fluctuations, phase stiffness $n_s/m^*$ is given by

$$n_s/m^* = \frac{2}{\pi e^2} (I_N - I_S)$$

where,

$$I_{N,S} = \int_0^\infty d\omega \sigma_1^{N,S}(\omega)$$

$$\sigma(\omega) = \left(\frac{2e^2}{\hbar c}\right) \left(\frac{\pi}{\omega}\right) \int_{-\infty}^{\infty} d\omega' [f(\omega') - f(\omega' + \omega)]$$

$$\times \left(\frac{1}{N}\right) \sum_{\vec{k}} \left(v_{k_x}^2 + v_{k_y}^2\right)$$

$$\left[N(\vec{k} + \vec{q}, \omega' + \omega) N(\vec{k}, \omega') + A_1(\vec{k} + \vec{q}, \omega' + \omega) A_1(\vec{k}, \omega')\right]$$

where, $v_{k_i} = \frac{\partial \varepsilon_{\vec{k}}}{\partial k_i}$ and $c$ is lattice constant along $z$-axis.

$$\left(\int_0^\infty d\omega \sigma_1(\omega) = \frac{\pi e^2 n}{2m^*} \ (f\text{-}\text{sum rule}) \text{ is used}\right)$$
Figure: Static superfluid density as a function of temperature for three values of the doping $x$. The solid curves are fits of power laws with logarithmic corrections. The intersection of $n_s(T)/m^*$ with the dashed line represents a simplified criterion for the \textit{BKT} transition temperature $T_c$. 
In the previous figure data are fits at a given doping level, in the form,

\[
\ln n_s(T)/m^* \approx a_0 + a_1 \ln(T_c^* - T) + a_2 \ln^2(T_c^* - T) + \ldots
\]

\[\Rightarrow\] a power law dependence close to \(T_c^*\) with logarithmic corrections.
Figure: Ratio of superfluid density to total hole density for the same doping values $x$ as in Fig. 1. The inset shows $\lambda^3(T = 0)/\lambda^3(T) = n_{s}^{3/2}(T)/n_{s}^{3/2}(T = 0)$, where $\lambda$ penetration depth, as a function of $(T_c^* - T)$. 

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Phase Diagram of Under-doped Cuprate Superconductors
Renormalization of phase stiffness due to phase(vortex) fluctuations:

- The *BKT* theory describes the unbinding of thermally generated pancake vortex-antivortex pairs.

- The relevant parameters are the dimensionless stiffness $K$ and the core energy $E_c$ of vortices. The $K$ is related to $n_s$ by

$$K(T) = \beta \hbar^2 \frac{n_s(T)}{m^*} \frac{d}{4}$$

$$\beta = \frac{1}{k_BT}, \ d \text{ is the average spacing between } CuO_2 \text{ layers.}$$

- The interaction between vortex-antivortex is given as,

$$V = 2\pi k_B T K \ln(r/r_0), \text{ where } r_0=\text{minimum pair size, twice of the vortex core radius } (\sim \xi_{ab} \text{ in plane, GL coherence length}).$$

- The $E_c$ is given as,

$$E_c = \pi k_B T K \ln \kappa, \text{ where } \kappa \text{ is Ginzburg parameter.}$$
Renormalization of phase stiffness due to phase(vortex) fluctuations contd..

1. Integrate out the vortex-antivortex pair starting from smallest pairs of size $r_0$, and their effect is taken as approximate renormalization of stiffness $K$ and fugacity $y = \exp (-\beta E_c)$.

2. In this way one get the “Kosterlitz recursion relations”,

$$\frac{dy}{dl} = (2 - \pi K)y$$

$$\frac{dK}{dl} = -4\pi^3 y^2 K^2$$

with $l = \ln(r/r_0$, logarithmic length scale.

3. (a) $T > T_c$: $K \rightarrow 0$ for $l \rightarrow \infty$, shows that the interaction is screened at large distances and the largest vortex-antivortex pairs unbind. The unbound vortices destroy the superconducting order and the “Meissner effect” and lead to dissipation.
Renormalization of phase stiffness due to phase(vortex) fluctuations contd..

(b) $T < T_c$: $K$ approaches to a finite value, $K^R \equiv \lim_{l \to \infty} K$ and $y \to 0$ for $l \to \infty$, shows that there are exponentially few large pairs and they still feel the logarithmic interaction. Bound pairs reduce $K$ and thus $n_s$, but do not destroy superconductivity. At $T_c$, $K^R$ jumps from a universal value of $2/\pi$ to zero.

- It turns out that the renormalization of $K$ for $T < T_c$ is very small so that one obtains $T_c$ from the simple criterion
  
  \[ K(T_c) = \frac{2}{\pi} \quad \text{or} \quad \frac{n_s(T_c)}{m^*} = \frac{2}{\pi} \frac{4k_B T_c}{\hbar^2 d} . \]

- The renormalized phase stiffness is given as,
  
  \[ \frac{n_s^R}{m^*} = \frac{4k_B T}{\hbar^2 d} K^R \]
Figure: Different temperature Scales of the cuprates as function of doping $x$. 

$T_c^*$ and $n_s/m^*$ are indicated in the graph.
Figure: Temperature $T^*$ at which a small suppression of the density of states at the Fermi energy “weak pseudogap” appears. The inset shows the suppression of the density of states (in arbitrary units) for $x = 0.155$ and $T = 4.5 T^*_c$ (solid line), $T = 2.3 T^*_c \approx T^*$ (dashed line), and $T = 1.01 T^*_c$ (dotted line).
Figure: Transition temperatures in the presence of a normal-state pseudogap. The inset shows the phase stiffness with and without pseudogap at $x = 0.122$. 
Phase stiffness for $\omega \neq 0$ (Dynamical case) and without phase fluctuations

Phase stiffness $n_s(\omega)/m^*$ is given for $\vec{q} \to 0$ as,

$$\frac{n_s(\omega)}{m^*} = \frac{1}{e^2} \omega \sigma_2^S(\omega)$$

where, the imaginary part $\sigma_2^S(\omega)$ of the dynamical conductivity is obtained from the FLEX approximation for the “dynamical current-current correlation function” using the “Kubo formula”.
Figure: Frequency-dependent phase stiffness $n_s(\omega)/m^*$ for doping $x = 0.122$ (underdoped) and at different temperatures.
Renormalized phase stiffness for $\omega \neq 0$ (Dynamical screening of vortex interaction):

- An applied electromagnetic field exerts a force on the vortices mainly by inducing a superflow, which leads to a Lorentz force on the flux carried by the vortices. A moving a vortex leads to dissipation in its core and thus to a finite diffusion constant $D_v$ which impedes its motion.

- If one assumes a rotating field of frequency $\omega$, small $VA$ pairs will rotate to stay aligned with the field. Large pairs, on the other hand, will not be able to follow the rotation and thus become ineffective for the screening.

- A pair can follow the field if its component vortex and antivortex can move a distance $2\pi r$ during one period $T_\omega = 2\pi /\omega$. During this time a vortex can move a distance of about the diffusion length $\sqrt{D_v T_\omega} = \sqrt{2\pi D_v /\omega}$, so that the critical scale for the pair size is given as,

$$r_\omega \equiv \sqrt{\frac{D_v}{2\pi \omega}}$$
Renormalized phase stiffness for $\omega \neq 0$ (Dynamical case) contd..

- Only vortex-antivortex pairs of size $r \leq r_\omega$ contribute to the screening. To avoid an unphysical kink in $n_s(\omega)/m^*$, a smooth cutoff $\bar{r}^2 = r_\omega^2 + r_0^2$ is used.

- To find the effect of phase (vortex) fluctuations on the phase stiffness, the “Kosterlitz recursion relations” are integrated numerically up to the cutoff $\bar{l} = \ln(\bar{r}/r_0)$, which depends on $D_v/r_0^2$, is given as,

$$D_v/r_0^2 \approx C_v \frac{k_B T}{\hbar} \exp \left( \frac{-E_0^p}{k_B T} \right)$$

where $C_v$ is dimensionless constant and $\frac{E_0^p}{k_B} = 1200 \text{ K}$ for 2212.
Figure: Renormalized phase stiffness $n_s^R(\omega)/m^*$ for doping $x = 0.122$ (underdoped) and at different temperatures. The unrenormalized stiffness is shown as dashed lines; these are the same data as in previous graph.
Figure: Renormalized phase stiffness $n_s^R(\omega)/m^*$ for $x = 0.122$ and $D_v/r_0^2 = 10^{17}\,s^{-1}$ as a function of temperature for different frequencies. The unrenormalized stiffness is shown as dashed lines. The dotted line represents the approximate criterion, for the $(\omega = 0)$ BKT transition.
Summary and concluding remarks

- The characteristic energy scales of hole-doped cuprate superconductors from a theory that includes both spin and Cooper-pair phase fluctuations are discussed. The former are described by the FLEX approximation, whereas the later are included by means of the “Berezinskii-Kosterlitz-Thouless” theory, taking the FLEX results as input.
- Vortices lead to the renormalization of the phase stiffness $n_s(\omega)/m^*$ to $n_s^R(\omega)/m^*$.
- The stiffness at $T \rightarrow 0$ shows a maximum at a doping level of $x = 0.2$, in good agreement with experiments. At the transition temperature $T_c$ the renormalized static phase stiffness $n_s(\omega = 0)/m^*$ vanishes, leading to the disappearance of the “Meissner effect”.

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Phase Diagram of Under-doped Cuprate Superconductors
The $T_c$ determined from spin and transverse phase (vortex) fluctuations shows the experimentally observed downturn in the underdoped regime and shows a maximum at optimum doping.

For $T_c < T < T_c^*$, where phase-coherent superconductivity is absent, phase fluctuations lead to a strong renormalization of $n_s/m^*$ at frequencies much smaller than $2\Delta_0$.

Local formation of Cooper pairs still takes place in this regime. This leads to a strong pseudogap of the same magnitude $\Delta_0$ and symmetry as the superconducting gap below $T_c$.

The frequency dependence of $n_s^R(\omega)/m^*$ at higher frequencies, $\omega \geq \Delta_0$, shows characteristic very similar to the superconducting phase. These features vanish only around $T_c^*$.
Summary and concluding remark contd..

- For $T_c^* < T < T^*$ there is a weak suppression in the density of states at the Fermi energy.

- In conclusion “exchange of spin fluctuations”, modified by “strong superconducting phase (vortex) fluctuations” in the underdoped regime, is the main mechanism of superconductivity in cuprates.
References

Onset of the vortexlike Nernst signal above $T_c$ in $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ and $\text{Bi}_{2}\text{Sr}_{2-y}\text{La}_y\text{CuO}_6$, Yayu Wang, Z. A. Xu, T. Kakeshita, S. Uchida, S. Ono, Yoichi Ando, and N. P. Ong, Phys. Rev. B 64, 224519 (2001).


References Contd...


Thank You