Review of Electrodynamics

First, the Questions

- **What is light?**
- **How does a butterfly get its colours?**

- **How do we see them?**

Plan of Review

- Electrostatics
- Magnetostatics
- Electrodynamics
- Electrodynamics in Matter
- Potentials
- Light
- And other things!

But first, some basics...

- Vector field $v(r)$ – a vector is associated with every point in space
- Divergence of a vector field – measure of “flux” $\nabla \cdot v$
- Gauss Divergence Theorem

$$\int_V \nabla \cdot \boldsymbol{v} dV = \int_S \boldsymbol{v} \cdot \boldsymbol{n} dS$$

- Curl of a vector field – measure of “vorticity” $\nabla \times v$
- Stokes Curl Theorem

$$\int_S (\nabla \times \boldsymbol{v}) \cdot \boldsymbol{n} dS = \int_L \boldsymbol{v} \cdot d\boldsymbol{l}$$

Electrostatics

- **Electric field of a point charge** q **is** $\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\left(\frac{\boldsymbol{r}}{r}\right)$
- **Force on another charge** q'**:** $\boldsymbol{F}_{elec} = q'\boldsymbol{E}$
- **A continuous distribution of charge** $\rho(\boldsymbol{r})$ **(Gauss Law)**

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$$

- **In electrostatics,** $\boldsymbol{\nabla} \times \boldsymbol{E} = 0$ **(Static electric fields lead to "conservative forces")**

Magnetostatics

- **Field field of a line element dl with current I**
  $\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\left(\frac{Id\boldsymbol{l} \times \boldsymbol{r}}{r^3}\right)$
- **Force on a charge q': $\boldsymbol{F}_{mag} = q'\boldsymbol{v} \times \boldsymbol{B}$**
- **A continuous static distribution of current distribution $\boldsymbol{j}(\boldsymbol{r})$ (Ampére's Law)**

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \boldsymbol{j}$$

- **$\boldsymbol{\nabla} \cdot B = 0$, ALWAYS! There are no magnetic monopoles!**

Electrodynamics

- **Changing magnetic fields “produce” electric fields (Faraday’s Law)**

$$\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0}$$

- **Changing electric fields produce magnetic fields (Maxwell’s modification to Ampére’s Law)**

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \boldsymbol{j} + \mu_0 \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$

And, Maxwell's Equations

- **In free space ($\rho = 0, \boldsymbol{j} = 0$), God said**

$$\begin{aligned} \boldsymbol{\nabla} \cdot \boldsymbol{E} &= 0 \\ \boldsymbol{\nabla} \cdot \boldsymbol{B} &= 0 \\ \boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} &= \boldsymbol{0} \\ \boldsymbol{\nabla} \times \boldsymbol{B} &= \mu_0 \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \end{aligned}$$

  and there was light!

- **Partial differential equations for six quantities (three components each of E and B)**
- **Solution? Not so bad as it seems!**

Maxwell's Equations in Matter

- **In matter ($\rho_f = 0, \boldsymbol{j}_f = \mathbf{0}$), God said**

$$
\begin{aligned}
\boldsymbol{\nabla} \cdot \boldsymbol{D} &= 0 \\
\boldsymbol{\nabla} \cdot \boldsymbol{B} &= 0 \\
\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} &= \mathbf{0} \\
\boldsymbol{\nabla} \times \boldsymbol{H} &= \frac{\partial \boldsymbol{D}}{\partial t}
\end{aligned}
$$

and there was light (with a different speed!!)!

- D – **Electric displacement**
- H – **Axillary field**

Material Properties

- **Relationship between electric displacement D and E (P–polarisation, ε–dielectric constant (material property))**

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \varepsilon \varepsilon_0 \boldsymbol{E}$$

- **Ferroelectricity – spontaneous P**
- **Relationship between axillary field H and B(M–magnetisation, χ–susceptibility (material property))**

$$\boldsymbol{H} = \frac{1}{\mu_0} \boldsymbol{B} - \boldsymbol{M}, \quad \boldsymbol{M} = \chi \boldsymbol{H}$$

- **Ferromagnetism – spontaneous M**

Back to Vacuum, Solution of Maxwell

- **Introduce potentials (ϕ – electric potential, $\boldsymbol{A}$ – magnetic vector potential)**

$$\begin{aligned} \boldsymbol{E} &= -\boldsymbol{\nabla}\phi - \frac{\partial \boldsymbol{A}}{\partial t} \\ \boldsymbol{B} &= \boldsymbol{\nabla} \times \boldsymbol{A} \end{aligned}$$

- **Coulomb Guage ($\phi = 0, \boldsymbol{\nabla} \cdot \boldsymbol{A} = 0$) leaves**

$$\nabla^2 \boldsymbol{A} = \frac{1}{c^2} \frac{\partial^2 \boldsymbol{A}}{\partial t^2}$$

the "Wave Equation"

- **Speed of light** $c = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}}$

And, out comes Light!

- **Look for wave like solutions** $A(\boldsymbol{r}, t) = \boldsymbol{A}_0 e^{(i\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)}$ **(**$\boldsymbol{k}(= \frac{2\pi}{\lambda}\hat{\boldsymbol{k}})$ **– wavevector,** λ **wavelength,** $\hat{\boldsymbol{k}}$ **– direction)**
- **Solution gives**

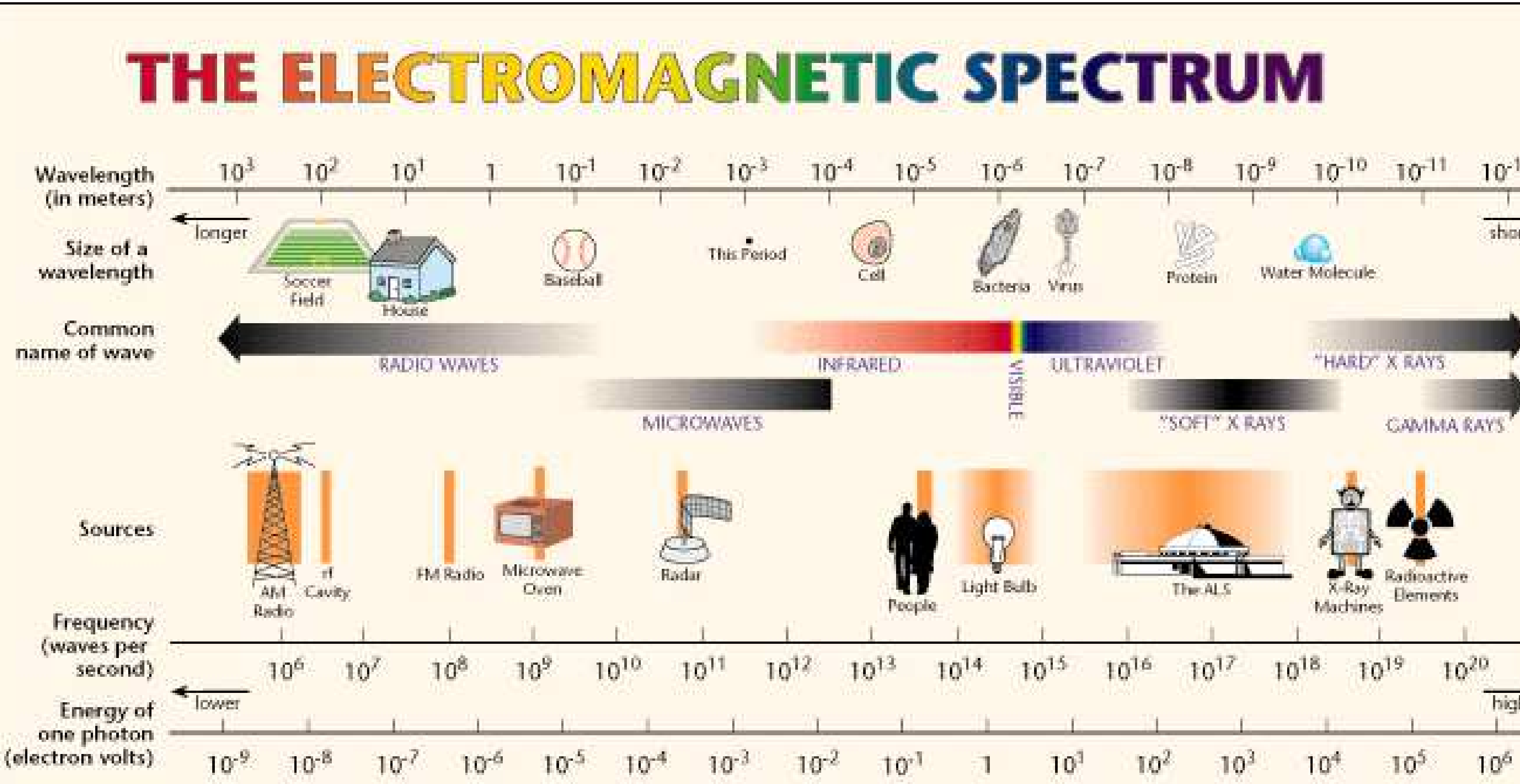
$$\omega^2 = c^2 k^2, \quad \boldsymbol{A}_0 \cdot \boldsymbol{k} = 0$$

- **Fields**

$$\begin{aligned} \boldsymbol{E} &= -\frac{\partial \boldsymbol{A}}{\partial t} = -i\omega \boldsymbol{A}_0 e^{(i\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)} \\ \boldsymbol{B} &= \boldsymbol{\nabla} \times \boldsymbol{A} = i\boldsymbol{k} \times \boldsymbol{A}_0 e^{(i\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)} \end{aligned}$$

- **Two possible polarisations; no longitudinal light waves!**

The Spectrum



One More Essential Thing! Charged Particle

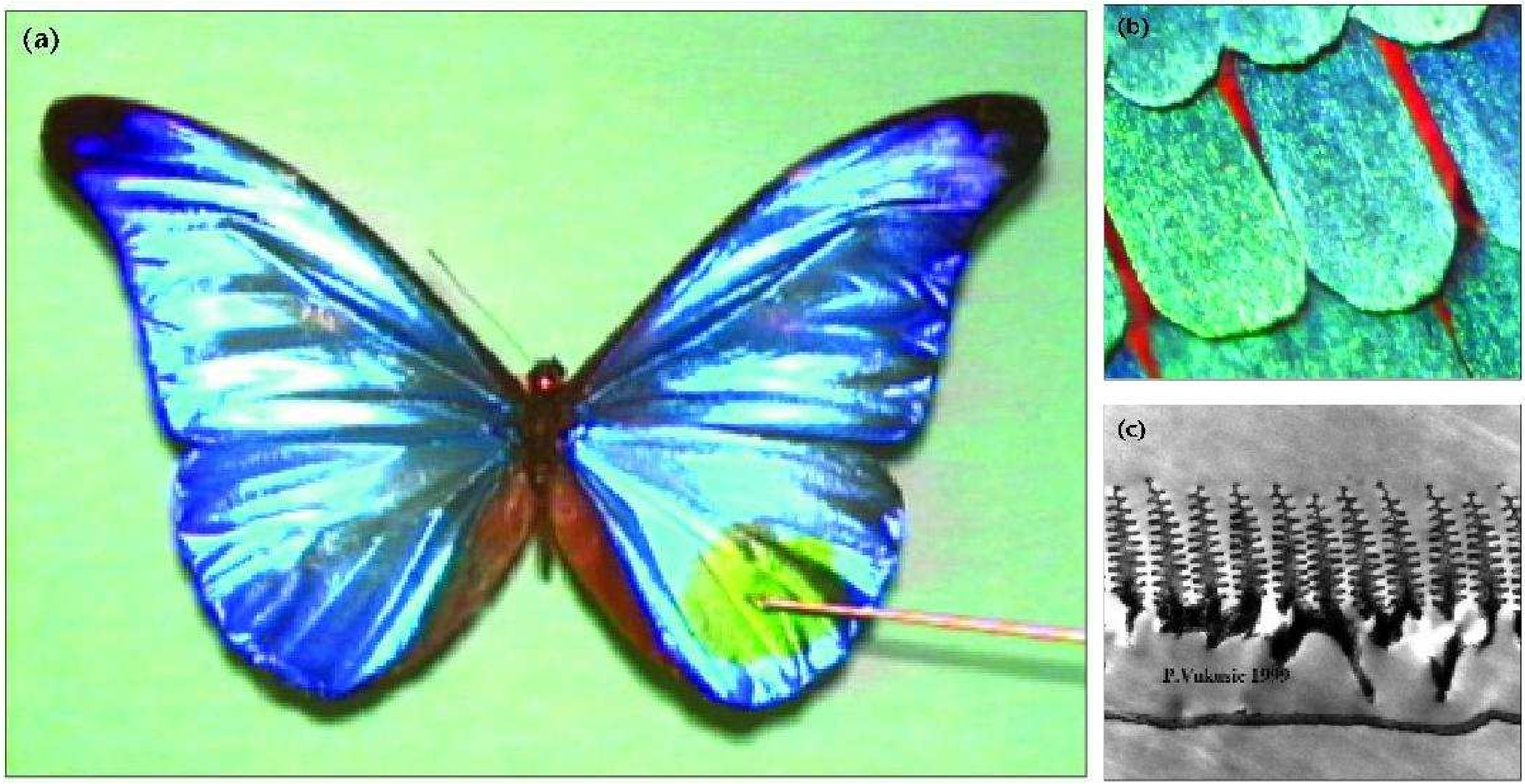
- **Hamiltonian of a charged particle (q) moving in an electromagnetic field**
- **FIeld described by $\phi(\boldsymbol{r})$ and $\boldsymbol{A}(\boldsymbol{r})$**
- **Hamiltonian**

$$H(\boldsymbol{r}, \boldsymbol{p}) = \frac{(\boldsymbol{p} - q\boldsymbol{A}) \cdot (\boldsymbol{p} - q\boldsymbol{A})}{2m} + q\phi$$

- **Useful in Quantum Mechanics!**
- **Derive the Lorentz force!**

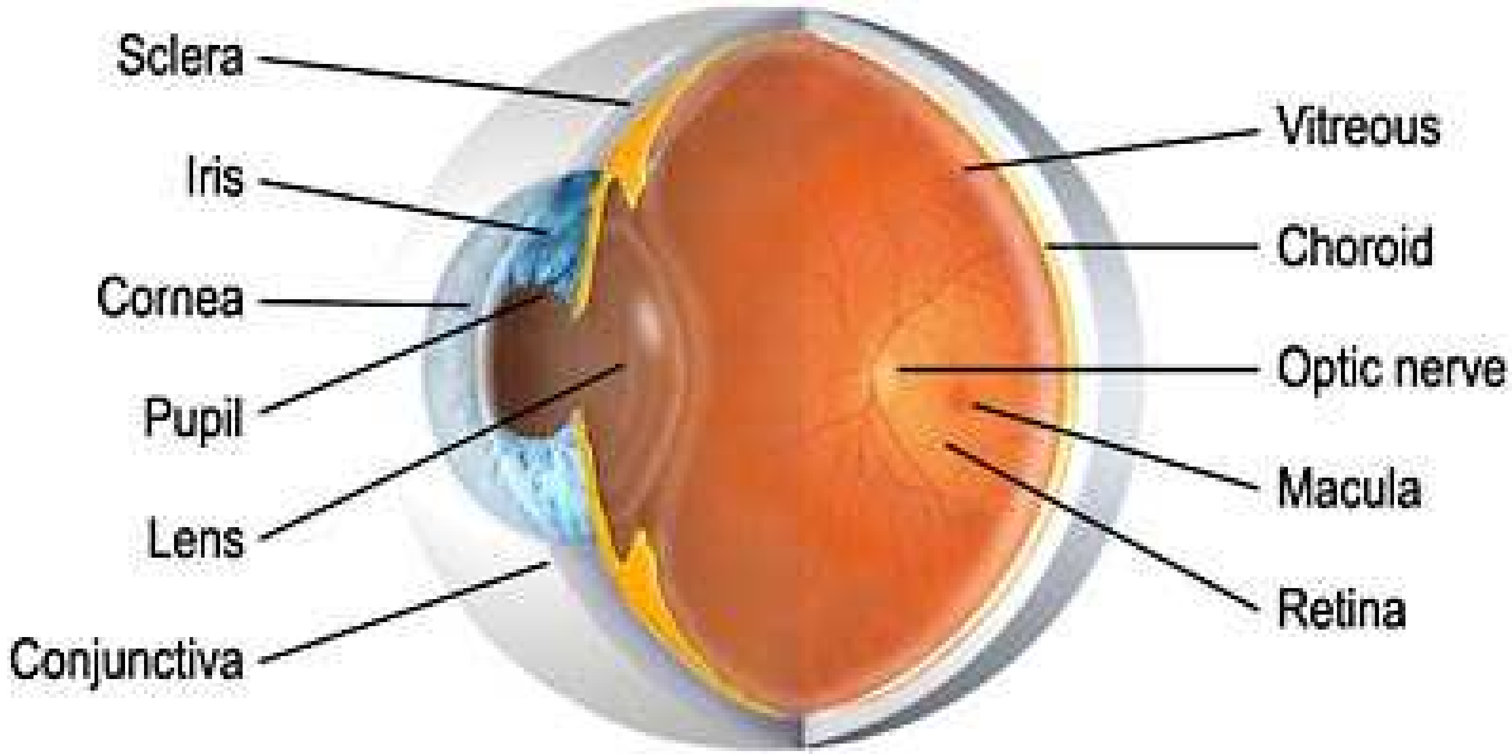
Colours of Butterfly...

- **Not really pigments! “Structural Colours”!!!**



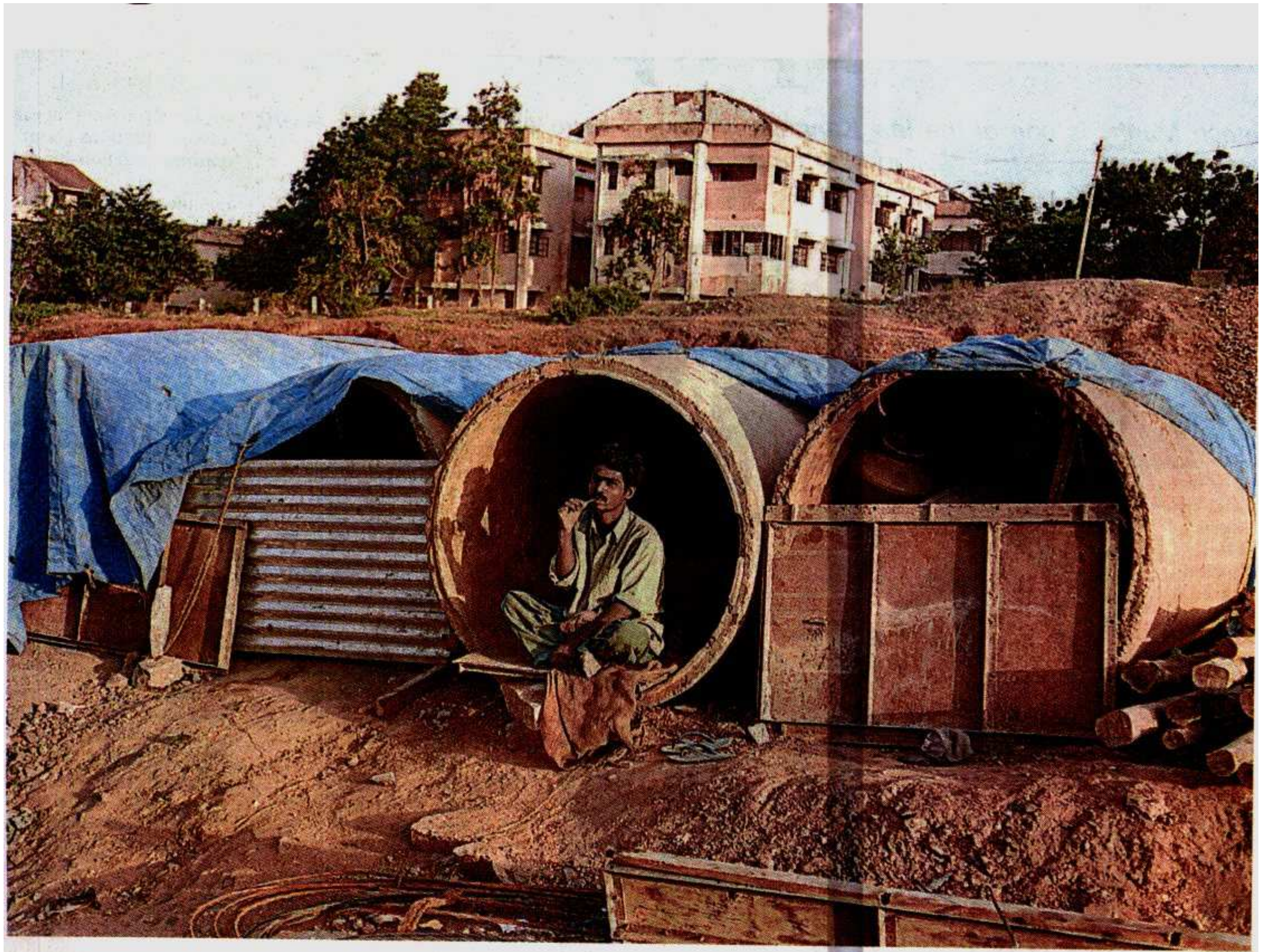
(Tayeb, Garlak, Enoch)

And, How do we see?



- **Rod cells, Cone cells**
- **Rhodopsin – Photoactive protein**
- **And, how does Sachin hit those straight drives?**

All good, buts lets not forget..



A home and a hearth still remain a pipe dream for him. — Photo: K. Gopinathan

Summary

- Electrodynamics, Maxwell's Equations
- Material Properties
- Wave type solutions
- Hamiltonian of charged particles